Exercise 37

Let $f(x) = cx + \ln(\cos x)$. For what value of c is $f'(\pi/4) = 6$?

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Solution

Start by taking the derivative of the function with respect to x.

$$f'(x) = \frac{d}{dx}[cx + \ln(\cos x)]$$

$$= \frac{d}{dx}(cx) + \frac{d}{dx}[\ln(\cos x)]$$

$$= c + \left[\frac{1}{\cos x} \cdot \frac{d}{dx}(\cos x)\right]$$

$$= c + \left[\frac{1}{\cos x} \cdot (-\sin x)\right]$$

$$= c + \left(-\frac{\sin x}{\cos x}\right)$$

$$= c - \tan x$$

Set $x = \pi/4$ to get $f'(\pi/4)$.

$$f'(\pi/4) = c - \tan\frac{\pi}{4} = c - 1$$

In order for $f'(\pi/4) = 6$,

$$c - 1 = 6$$

$$c = 7.$$